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Structures of rotating traditional neutron stars and hyperon stars in the relativistic σ-ω model

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Abstract. The influence of rotation on the total masses and radii of neutron stars is calculated by Hartle's slow-rotation formalism, while the equation of state is considered in a relativistic σ-ω model. As the changes of the mass and radius of a real neutron star caused by rotation are very small in comparison with the total mass and radius, one can see that Hartle's approximate method is rational to deal with the rotating neutron stars. If three property values, mass, radius and period, are observed for the same neutron star, then the EOS of this neutron star could be decided entirely.

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1 Introduction

Neutron stars are dense, neutron-packed remnants of massive stars after their supernova explosions. Recently, in both experiment and theory, much interest is focused on neutron stars mainly for two reasons [1–3]. One is to determine the equation of state (EOS) of superdense matter, and then to understand the early universe, its evolution to the present day and the various astrophysical phenomena; the other one is that the neutron star is one of the most probable sources of detectable gravitational waves. To understand neutron stars, the first thing to do is to understand their structure, such as the compositions, total masses, radii, redshifts etc. Because of their strongly gravitational field and high density, neutron stars must be studied in the framework of general relativity. For a static neutron star, giving the EOS, using TOV equations [4], which are deduced from the Einstein field equations, one can get an exact solution. But for a rotating neutron star, the components of the Einstein field equations become much more difficult. Nowadays, several approximate solutions of this problem have been developed [5,6], among which Hartle's slow-rotation formalism [7,8] is the most popular one.

The properties of neutron stars such as masses, rotational frequencies, radii, moments of inertia and redshifts are sensitive to the EOS of the matters [2,3]. As the interior core contains most of the mass of a neutron star, attention to the EOS is mostly focused on the neutron star core, that is on the matters at a density several times above the nuclear matter saturation density. In the core of a neutron star, the compositions still remain blurry in some degree due to the high density and uncertainty of strong interaction, but as density increases in the neutron star, neutrons can drip out of nuclei and form a neutron gas; and due to the chemical equilibrium and the electric charge neutrality, there are at least protons and electrons in neutron stars; on the other hand, as the density increases, hyperons will be dominant in neutron stars [9]. In this paper, Hartle's formalism will be used to deal with two kinds of rotating neutron stars: the traditional neutron stars, in which n, p, e, μ, are the main elements; and hyperon stars, in which $n, p, e, \mu, \Lambda, \Sigma, \Xi, \Delta$ are the main elements (in fact, the compositions of the neutron star may be much more complex than this). Their EOSs will be considered in the relativistic σ-ω model.

In this paper, we adopt the metric signature $-+++$, $G = c = 1$.

2 Hartle's slow-rotation formalism

In relativity, the space-time geometry of a rotating star in equilibrium is described by a stationary and axisymmetric metric of the form

$$\mathrm{d}s^2 = -e^{2\nu}\mathrm{d}t^2 + e^{2\lambda}\mathrm{d}r^2 + e^{2\psi}(\mathrm{d}\phi - \omega\mathrm{d}t)^2 + e^{2\mu}\mathrm{d}\theta^2\,, \quad (1)$$

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where $\omega(r)$ is the angular velocity of the local inertial frame and is proportional to the star's rotational frequency Ω, which is the uniform angular velocity of the star relative to an observer at infinity. Expanding the metric function at second order in Ω, one has [7]

$$e^{2\nu} = e^{2\varphi}\Big[1 + 2(h_0 + h_2 P_2)\Big], \quad (2)$$

$$e^{2\lambda} = \left[1 + \frac{2}{r}(m_0 + m_2 P_2) \times \left(1 - \frac{2M_0(r)}{r}\right)^{-1}\right]\left(1 - \frac{2M_0(r)}{r}\right)^{-1}, \quad (3)$$

$$e^{2\psi} = r^2 \sin^2\theta\Big[1 + 2(v_2 - h_2)P_2\Big], \quad (4)$$

$$e^{2\mu} = r^2\Big[1 + 2(v_2 - h_2)P_2\Big], \quad (5)$$

where $e^{2\varphi}$ and $M_0(r)$ denote the metric function and the mass of the non-rotating neutron star with the same central density, respectively; P_2 is the Legendre ploynomial of order 2; the perturbation functions m_0, m_2, h_0, h_2, v_2 are proportional to Ω^2 and are to be calculated from the Einstein field equations.

From the (t, ϕ) component of the Einstein field equations, one gets [7]

$$\frac{1}{r^4}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^4 j \frac{\mathrm{d}\bar{\omega}}{\mathrm{d}r}\right) + \frac{4}{r}\frac{\mathrm{d}j}{\mathrm{d}r}\bar{\omega} = 0, \quad (6)$$

where $j(r) = e^{-\varphi}[1 - 2M_0(r)/r]^{\frac{1}{2}}$, $\bar{\omega} = \Omega - \omega$, which denotes the angular velocity of the fluid relative to the local inertial frame. The boundary conditions are imposed as $\bar{\omega} = \bar{\omega}_c$ at the center, $\frac{\mathrm{d}\bar{\omega}}{\mathrm{d}r}|_{\bar{\omega}_c=0}$, where $\bar{\omega}_c$ is chosen arbitrarily. Integrating eq. (6) outward from the center of the star, one would get the function $\bar{\omega}(r)$. Outside the star, from eq. (6) one has $\bar{\omega}(r) = \Omega - \frac{2J}{r^3}$, where J is the total angular mementum of the star, which takes the form [2] $J = \frac{1}{6}R_0^4 \frac{\mathrm{d}\bar{\omega}}{\mathrm{d}r}|_{r=R_0}$. Thus at the surface, one can determine the angular velocity Ω corresponding to $\bar{\omega}_c$ as

$$\Omega = \bar{\omega}(R_0) + 2\frac{J}{R_0^3}. \quad (7)$$

In Newton's theory, there exists a maximal rotating frequency to the star formed by a perfect fluid, at which there just comes into being a balance of gravitational and centrifugal forces at the star's equator. Exceeding the maximal frequency, the star will engender mass shedding at the equator. According to Newton's mechanics and Newton's gravitational theory, one can get the maximal frequency as

$$\Omega_c = \sqrt{\frac{M}{R}}, \quad (8)$$

in which, M, R are the mass and the radius of the star, respectively. In the framework of general relativity, the upper limit of the frequency, which is called Kepler frequency, is the solution of the following equation [10]:

$$\Omega_k = e^{\upsilon-\psi}\sqrt{\frac{\upsilon'}{\psi'} + \left(\frac{\omega'}{2\psi'}e^{\psi-\upsilon}\right)^2} + \frac{\omega'}{2\psi'} + \omega. \quad (9)$$

From the (t,t) and (r,r) components of the Einstein field equations, one gets two coupled ordinary differential equations of m_0 and h_0 as [7,8]

$$\frac{\mathrm{d}m_0}{\mathrm{d}r} = 4\pi r^2 \frac{\mathrm{d}(p+\rho)}{\mathrm{d}p}(\rho + p)p_0^* + \frac{1}{12}j^2 r^4\left(\frac{\mathrm{d}\bar{\omega}}{\mathrm{d}r}\right)^2 - \frac{1}{3}r^3\frac{\mathrm{d}(j^2)}{\mathrm{d}r}\bar{\omega}^2, \quad (10)$$

$$\frac{\mathrm{d}p_0^*}{\mathrm{d}r} = -\frac{m_0(1+8\pi r^2 p)}{[r - 2M_0(r)]^2} - \frac{4\pi r^2(p+\rho)}{r - 2M_0(r)}p_0^* + \frac{1}{12}\frac{r^4 j^2}{r - 2M_0(r)}\left(\frac{\mathrm{d}\bar{\omega}}{\mathrm{d}r}\right)^2 + \frac{1}{3}\frac{\mathrm{d}}{\mathrm{d}r}\left[\frac{r^3 j^2 \bar{\omega}^2}{r - 2M_0(r)}\right], \quad (11)$$

where $p_0^* = -h_0 + \frac{1}{3}r^2 e^{-2\upsilon}\bar{\omega}^2 + C$; here C is a constant determined by the demand that h_0 be continuous across the star's surface. These equations are also integrated outward, with the boundary conditions that both m_0 and p_0^* vanish at the origin. With the same central density, the difference between the mass of the rotating star and the non-rotating star is

$$\delta M = m_0(R_0) + \frac{J^2}{R_0^3}. \quad (12)$$

The difference of the mean radius is

$$\delta r = -p_0^*(\rho + p)\frac{\mathrm{d}p}{\mathrm{d}r}. \quad (13)$$

3 Model for the EOSs —the relativistic σ-ω model

There are several models to deal with superdense matters, such as non-relativistic models and relativistic field-theoretical models [3]. In the different models, the fractions of particles in superdense matters are different, and then the bulk properties of superdense matters are different, that is, their EOSs are different. Here the relativistic σ-ω model will be adopted [11]. The Lagrangian density of this model is

$$L = \sum_B \bar{\psi}_B\left(i\gamma_\mu\partial^\mu + m_B - g_{\sigma B}\sigma - g_{\omega B}\gamma_\mu\omega^\mu - \frac{1}{2}g_{\rho B}\gamma_\mu\vec{\tau}\cdot\vec{\rho}^\mu\right)\psi_B + \frac{1}{2}(\partial\sigma)^2 - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - U(\sigma) - \frac{1}{4}\vec{\rho}_{\mu\nu}\cdot\vec{\rho}^{\mu\nu} + \frac{1}{2}m_\rho^2\vec{\rho}_\mu\cdot\vec{\rho}^\mu + \sum_l \bar{\psi}_l(i\gamma_\mu\partial^\mu - m_l)\psi_l \quad (14)$$

in which $U(\sigma) = a\sigma + \frac{1}{3!}c\sigma^3 + \frac{1}{4!}d\sigma^4$, $F_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu$, $\vec{\rho}_{\mu\nu} = \partial_\mu\vec{\rho}_\nu - \partial_\nu\vec{\rho}_\mu$, ψ_B is the field operator of baryon B ($B = n, p$ for the traditional neutron stars, $B = n, p, \Lambda, \Sigma, \Xi, \Delta$ for the hyperon stars); ψ_l is the field operator of lepton l ($l = e, \mu$); and σ, ω^μ, $\vec{\rho}^\mu$ are the field operators of the σ-, ω-, ρ-meson, respectively; $g_{\sigma B}$, $g_{\omega B}$, $g_{\rho B}$ are the

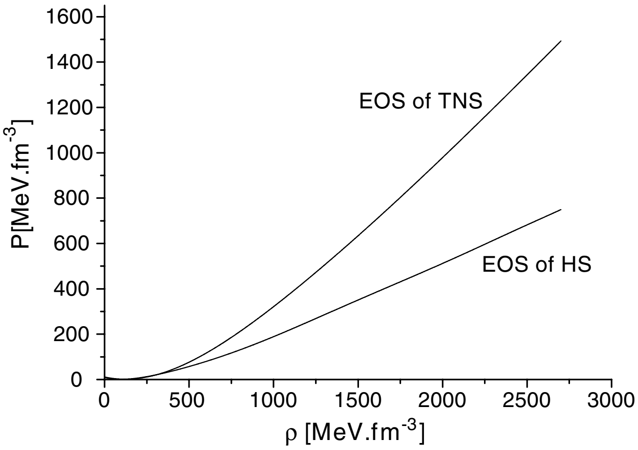

Fig. 1. Equation of state of traditional neutron stars (TNS) and hyperon stars (HS).

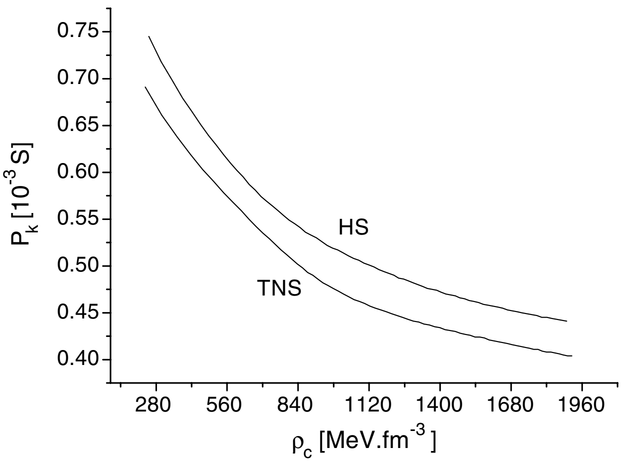

Fig. 2. The minimum rotational periods as a function of the central density of TNS and HS when they rotate at their Kepler frequency.

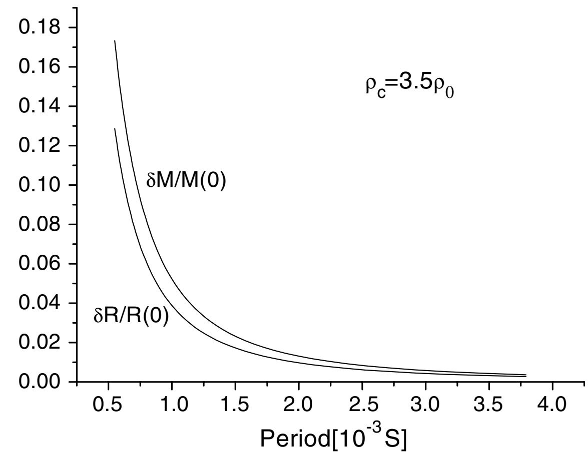

Fig. 3. Increment of the mass and radius of TNS compared to that of non-rotating TNS at the central density $\rho_c = 3.5\rho_0$, where ρ_0 is the saturation density of nuclear matters.

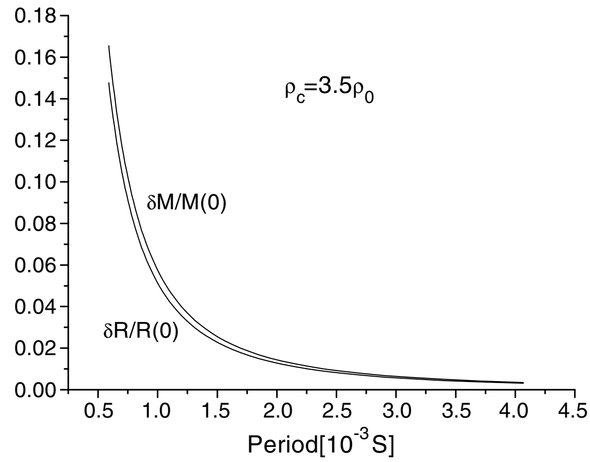

Fig. 4. Increment of the mass and radius of HS compared to that of non-rotating HS at the central density $\rho_c = 3.5\rho_0$.

coupling constants between σ-, ω-, ρ-meson and baryon B, respectively. In general, the coupling constants between σ (ω or ρ) meson and neutron and proton are equal and decided by the saturated property of nuclear matter ($g_{\sigma n} = g_{\sigma p}, g_{\omega n} = g_{\omega p}$) and the symmetry energy of nuclear matter ($g_{\sigma\rho}, g_{\omega\rho}$); $m_B, m_l, m_i, (i = \sigma, \omega, \rho)$ are the mass of baryon, lepton, meson, respectively; $\vec{\tau}$ is the isospin operator. As to leptons, we assume they are free Fermi gas. From this Lagrangian density, we can obtain the EOS of superdense matters, $p = p(\rho)$, in which, p and ρ are the pressure and energy density of superdense matters, respectively. In the 1-loop approximation, the loop's contribution to the propagators of the nucleon and σ-meson is considered, and the renormalization is used to renormalize the divergent part of the loop contribution (for neutron star matters, the effect of considering the 1-loop approximation is not very obvious). In the numerical calculation, we adopt the following parameter values [11]: $a = -2.1 \times 10^7\,\mathrm{MeV}^3$, $c = 0.97 \times M_n$, $d = 1277$, $g_s = 6.73$, $g_v = 8.59$, $M_n = 938\,\mathrm{MeV}$, $m_\omega = 783\,\mathrm{MeV}$, $m_\sigma = 550\,\mathrm{MeV}$, $m_\rho = 770\,\mathrm{MeV}$, and the incompressibility of nuclear matter is 224 MeV, which is consistent with the experiment result [12,13]. From fig. 1, it is clear that the EOS of traditional neutron stars is stiffer than the EOS of hyperon stars.

4 Numerical results and discussion

In order to compare numerical results with observation, we present some typical observation values here. As we know, only a few masses have been determined in the observation of more than a thousand neutron stars. There are two typical observed mass data: $1.36 \pm 0.08 M_\odot$ [14] for double-neutron-star binaries, and $1.87^{+0.23}_{-0.17} M_\odot$ (Vela X-1) [15] and $1.8 \pm 0.4 M_\odot$ (Cygnus X-2) [16] for X-ray binaries.

The numerical results for the rotating traditional neutron stars and hyperon stars in the relativistic σ-ω model are shown in the following figures and table.

Figure 2 gives the minimum rotational periods as a function of the central density of traditional neutron stars and hyperon stars when they rotate at their Kepler

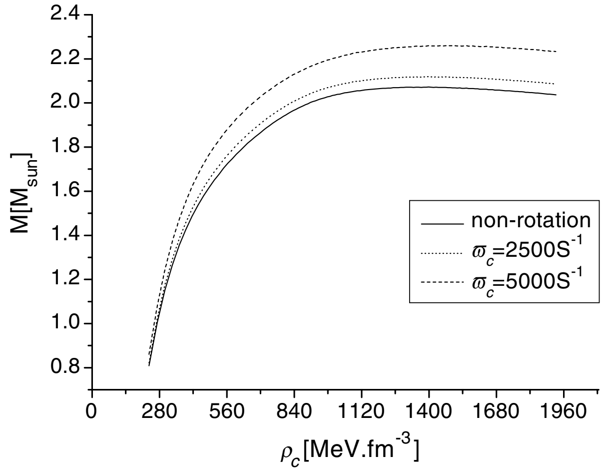

Fig. 5. The total mass of non-rotating and rotating TNS as a function of the central density, where the mass is in units of solar masses.

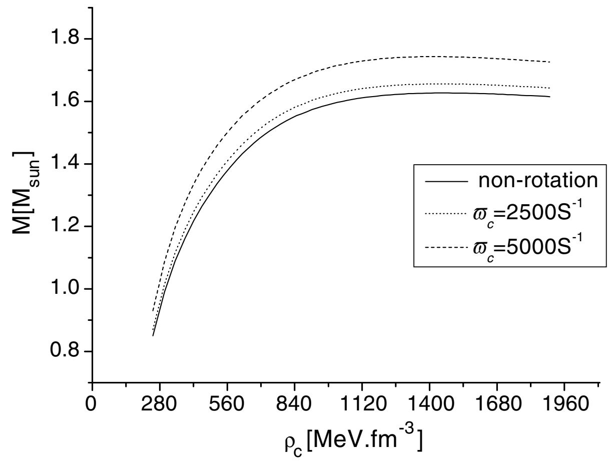

Fig. 7. The total mass of non-rotating and rotating HS as a function of the central density, where the mass is in units of solar masses.

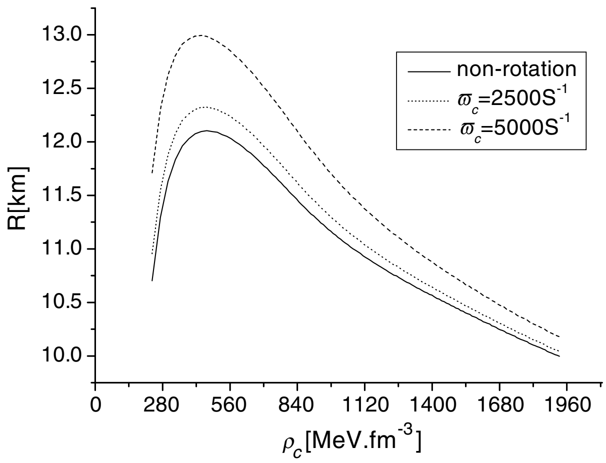

Fig. 6. The radius of non-rotating and rotating TNS as a function of the central density.

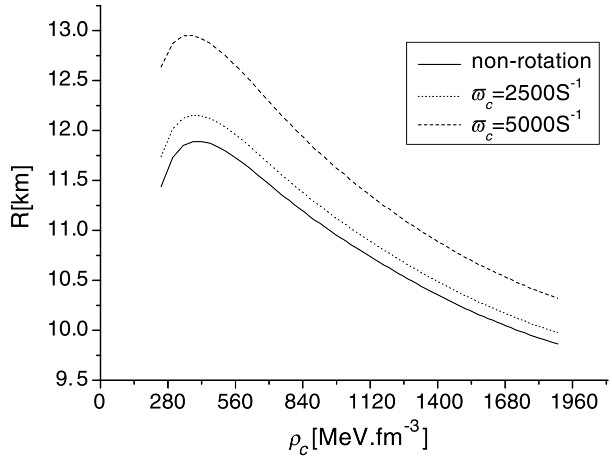

Fig. 8. The radius of non-rotating and rotating HS as a function of the central density.

frequency. From this figure one can see that all the calculated minimum periods are smaller than the minimum observational value of the periods of the pulsars, 1.6 ms [3]; from this fact, one can say that the adopted EOS may be a rational one to describe neutron star matters. Comparing fig. 2 with the following figures, it is easy to see that all the following figures are calculated with rotating frequencies smaller than the Kepler frequency.

Figures 3-4 show the changes of masses and radii between rotating neutron stars and non-rotating neutron stars as a function of the rotational period. At a given central density, it is easy to see that, as the neutron stars rotate slowly, the increment of the masses and radii will reduce sharply when the neutron stars' rotational periods are smaller than 1 ms. But as the periods become bigger than 1.6 ms, which is the period of the fastest rotating pulsars in observation [3], the change of the increment of the masses and radii caused by the rotation will become very weak, and the increment will not be higher than 2%. As the changes of the mass and radius of a real neutron star caused by rotation are very small in comparison with the total mass and radius, we can see that Hartle's approximate method is rational.

Figures 5-8 show the masses and radii of rotating and non-rotating traditional neutron stars and hyperon stars as a function of the central density at a given central angular velocity relative to the local inertial frame. From these figures, one can see that around the typical observational radii with value of 12 km, the increments of the radii are bigger than the increments of other radii. In table 1, some typical values of these figures are listed. For the hyperon stars, the EOS is so soft that, even for periods smaller than the smallest observational value, the total mass cannot reach the observation value of X-ray binaries, that is about $1.8M_\odot$.

Figure 9 shows the period as a function of the central density at two given central angular velocities relative to the local inertial frame. From this figure, one can see that a bigger central density and a bigger central angular velocity relative to the local inertial frame correspond with a smaller period. In fig. 9 one can find out that as the EOS of

Table 1. Rotating neutron star's properties in the relativistic σ-ω model. TNS and HS denote traditional neutron stars and hyperon stars, respectively; $\bar{\omega}_c$ is the angular velocity relative to the local inertial frame at the center, with units of $(10^3\,\mathrm{s}^{-1})$; ρ_c is the central density, with units of $(10^{18}\,\mathrm{kg}\cdot\mathrm{m}^{-3})$; R_0 and M_0 denote the radius and the mass of the non-rotating stars, R and M denote the radius and the mass of the rotating neutron stars, respectively; the unit of mass is the solar mass $(M_\odot)$, $\frac{\delta R}{R_0}$ and $\frac{\delta M}{M_0}$ are the fractional difference of the radius and mass between rotating neutron stars and non-rotating neutron stars; P is the rotational period.

	$\bar{\omega}_c$	ρ_c	R_0 (km)	M_0	R (km)	M	$\frac{\delta R}{R_0}$	$\frac{\delta M}{M_0}$	P (ms)
TNS	2.50	1.067	11.98	1.768	12.18	1.808	0.017	0.023	1.488
	5.00	0.903	10.21	1.650	12.94	1.801	0.070	0.092	0.791
HS	2.50	0.921	11.80	1.336	12.04	1.367	0.020	0.023	1.686
	5.00	0.811	10.87	1.265	12.87	1.385	0.084	0.095	0.873

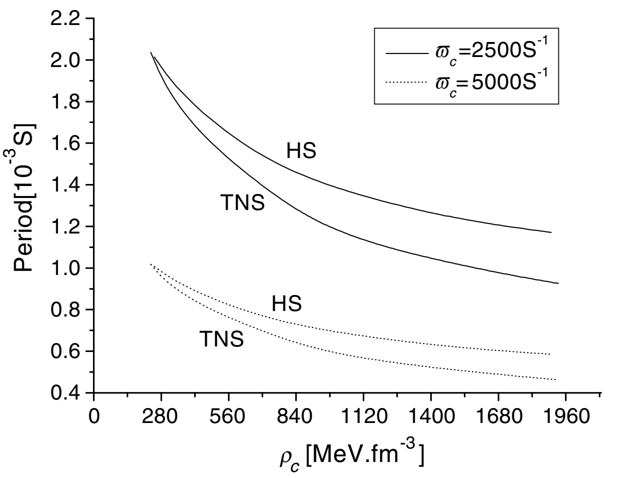

Fig. 9. Period as a function of the central density at two different angular velocities relative to the local inertial frame at the center.

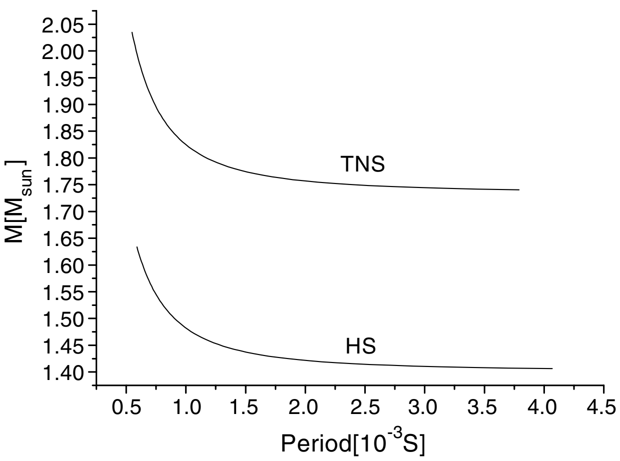

Fig. 10. The masses as a function of the period at $\rho_c = 3.5\rho_0$ for TNS and HS.

hyperon stars is softer, at the same conditions, the period of hyperon stars is appreciably bigger than the period of traditional neutron stars. Figures 10-11 show the masses and radii of the traditional neutron stars and hyperon stars as a function of the period. One can see that even the central density of the hyperon stars is bigger than those

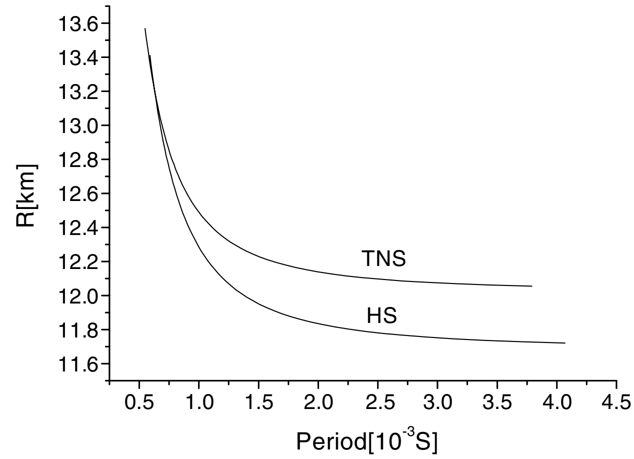

Fig. 11. The radii as a function of the period at $\rho_c = 3.5\rho_0$ for TNS and HS.

of the traditional neutron stars, the masses and radii of the hyperon stars are smaller than those of the traditional neutron stars. Another interesting result is that at a given central density, when the period increases to a value which is bigger than the smallest observational period, 1.6 ms, the change of the masses and the radii with the period is not obvious, which is consistent with the result of fig. 3.

From figs. 2-11, one can see that at a given period, the central density and the central angular velocity relative to the local inertial frame could be chosen freely, but if the masses and the period of a neutron star are given (by the observational value) at the same time, then the central density and the central angular velocity relative to the local inertial frame will be decided. As we know, there is another observational value: the radius of the neutron star, the decided central density and central angular velocity do not always give the exact radius. So one can say that if these three observational values, mass, radius and period are given to the same neutron star, there is only one special EOS that could give a set of calculated values, which fit like a glove to the set of observational values, that is, these three observational values could decide the EOS entirely. But the problem is that there is no neutron star with both mass and radius observationally determined up to the present.

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